Chapter test.

a) ${}_{10}C_7 + {}_{10}C_8$.

Values of ${}_nC_r$ from the pattern creating Pascal's triangle,
Hence. ${}_nC_r = t_{n,r}$.

$\therefore \; {}_{10}C_7 + {}_{10}C_8 = {}_{10+(8-7)}C_8$

$= {}_{10+1}C_8$

$= {}_{11}C_8$.

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a) $(3x-4)^4$.

let $(3x-4)^4 = (a+b)^4$.

$a = 3x$.

$b = -4$.

Using the pascal's triangle the terms in row 4 for exponent 4 are. 1, 4, 6, 4 and 1. Thus, substitute 3x for a and −4 for b.

$(3x-4)^4 = 1(3x)^4 + 4(3x)^3(-4) + 6(3x)^2(-4)^2 + 4(3x)(-4)^3 + 1(-4)^4$.

$= 81x^4 + 4(27x^3)(-4) + 6(9x^2)(16) + 4(3x)(-64) + 256$.

$= 81x^4 - 432x^3 + 864x^2 - 768x + 256$.

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4. a) $(8x-3)^5$.

let $(8x-3)^5 = (a+b)^5$, $a = 8x$, $b = -3$.

$(a+b)^n = {}_nC_0 a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^r + \dots + {}_nC_n b^n$.

Substitute 8x for a and −3 for b.

$(8x-3)^5 = \sum_{r=0}^{5} {}_5C_r (8x)^{5-r} (3^r)$

$= {}_5C_0(8x)^5 + {}_5C_1(8x)^4(-3) + {}_5C_2(8x)^3(-3)^2 + {}_5C_3(8x)^2(-3)^3 + {}_5C_4(8x)(-3)^4 + {}_5C_5 \dots$

$= 32768x^5 - 61440x^4 + 46080x^3 - 17280x^2 + 3240x - 243$.

6 b) 3 members out of 20, Permutation is concidered.

$$_{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \times 19 \times 18 \times 17!}{17!} = 20 \times 19 \times 18$$

$= 6840$ ways.

a) Choosing a starter, a marshal and a timer, combination is used.

$$_{20}C_3 = \frac{20!}{3!(20-3)!} = \frac{20!}{3!\,17!} = \frac{20 \times 19 \times \overset{3}{\cancel{18}} \times 17!}{\cancel{3 \times 2} \times 1 \times 17!} = 20 \times 19 \times 3.$$

$= 1140$ ways.

c) Answers in a) and b) should not be the same because in a combination, the order of selection of subsets is not a factor whereas in permutation, the order of selection is a factor.

5.1 Organized Counting with Venn Diagrams.

1 a) $A \cup B$ refers to all elements contained in set A or B.

$\Rightarrow$ R2, R3, R5, R6, R8.

b) $B \cap C$. The intersection of B and C refers to all elements contained in set B and C.

$\Rightarrow$ R7, R8.

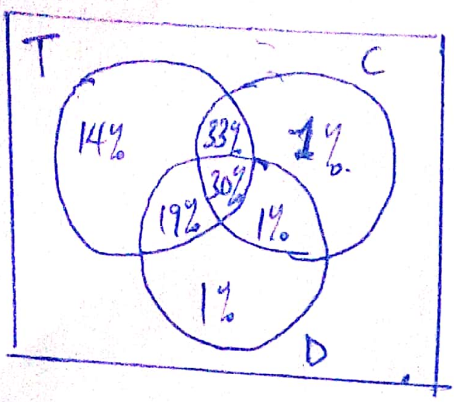
c) $A \cap C$. The Intersection of A and C refers to all elements in A that are also in C.

$\Rightarrow$ R5, R8.

d) $B \cup S$. The union of B and S refers to all elements in set B or in set S.

$\Rightarrow$ R3, R6, R7, R8, R1.

3. Let T represent colour televisions, C computers and D dishwashers.




C and D = 31% − 30% = 1%.

T and C = 63% − 30% = 33%.

T and D = 49% − 30% = 19%

T, C and D = 30%.

T only = 96% − (33+19+30)%
= 96% − 82%
= 14%.

C only = 65% − (33+30)%
= 65% − 64%
= 1%.

D only = 51% − (19+30+1)%
= 51% − 50%
= 1%.

a) Households not included in the survey

Total − Sum of survey in T only, C only, D only − Sum of survey in $(T\cap C), (T\cap D), (D\cap C), (T\cap C\cap D)$

= 100 − (14+1+1) − (30 + 33 + 19 + 1)

= 100 − 16 − 83

= 100 − 99

= 1% of households were not included in these survey results.

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5.2

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

4. i) ${}_{15}C_{11}$

$${}_{15}C_{11} = \frac{15!}{11!(15-11)!} = \frac{15!}{11!\,4!}$$

$$= \frac{15\times \cancel{14}^{7}\times 13\times \cancel{12}\times \cancel{11!}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1\times\cancel{11!}} = 15\times 7\times 13$$

$$= 1365.$$

5.3

10. If all 5 of the first 5 questions are answered, then there are ${}_7C_3$ ways to choose the other 3 questions answered.

$${}_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!} = 35 \text{ ways.}$$

If 4 out of 5 questions are answered, there are ${}_5C_4$ ways.

$${}_5C_4 = \frac{5!}{4!} = \frac{5\times4!}{4!} = 5 \text{ ways.}$$

ways to pick 8 questions $= 5\times35 = 175$ ways.

ways to pick 8 out of 12 questions $= 35+175 = 210$ ways.

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12. ${}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$.

$$= \frac{(15\times14\times13\times12\times11)}{(1\times2\times3\times4\times5)} \times \frac{(10\times9\times8\times7\times5)}{(1\times2\times3\times4\times5)} = 3003\times252 = 756756 \text{ ways.}$$

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5.4

15 a) $(x+y)^6$

let $(x+y)^6 = (a+b)^6$, $a = x$, $y = b$.

Using the pascal's triangle the terms in row 6 for exponent 6 are 1, 6, 15, 20, 15, 6 and 1. Substitute x for a and y for b.

$$(x+y)^6 = 1(x^6) + 6(x^5y) + 15(x^4y^2) + 20(x^3y^3) + 15(x^2y^4) + 6xy^5 + y^6.$$
$$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6.$$

19. $\left(\frac{1}{x^2}+2x\right)^5$

terms for row 5 for exponent 5 in the Pascal's triangle are. 1, 5, 10, 10, 5, 1

Concider $(a+b)^5$

$\left(\frac{1}{x^2}+2x\right)^5 = (a+b)^5$

$\Rightarrow a = \frac{1}{x^2}, \quad b = 2x.$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Substitute $\frac{1}{x^2}$ for a and 2x for b.

$\left(\frac{1}{x^2}+2x\right)^5 = 1\left(\frac{1}{x^2}\right)^5 + 5\left(\frac{1}{x^2}\right)^4(2x) + 10\left(\frac{1}{x^2}\right)^3(2x)^2 + 10\left(\frac{1}{x^2}\right)^2(2x)^3 +$

$5\left(\frac{1}{x^2}\right)(2x)^4 + (2x)^5$

$= \frac{1}{x^{10}} + \frac{10}{x^7} + \frac{40}{x^4} + \frac{80}{x} + 80x^2 + 32x^5.$

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23. $1024x^{10} - 3840x^8 + 5760x^6 - 4320x^4 + 1620x^2 - 243.$

Since there are 6 terms, the exponend is 5.

The first term. $1024x^{10} = (4x^2)^5 \Rightarrow a = 4x^2.$

The last term $243 = b^5.$

$3^5 = b^5$

$\Rightarrow b = 3.$

expansion.

Concider the binomial $(a-b)^5 \Rightarrow (4x^2-3)^5$

# The Binomial Theorem

1. e) $(a+b)^5$

Using the Pascal's triangle, the coeficients in row 5 for exponent 5 are 1, 5, 10, 10, 5, 1

$$(a+b)^5 = a^5 + a^4 b + a^3 b^2 + a^2 b^3 + ab^4 + b^5$$

Substitute the coeficients.

$$(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5.$$

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4. $(a+b)^6$.

number of terms = exponent + 1
= 6+1
= 7 terms

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Apply, Solve, Communicate.

5 a) ${}_9C_0 + {}_9C_1 + \dots + {}_9C_9$.

Values of ${}_nC_r$ form the pattern creating Pascal's triangle.

$${}_nC_r = t_{n,r}.$$

$${}_9C_0 + {}_9C_1 + \dots + {}_9C_9 = 2^9$$
$$= 512.$$

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6. $\sum\limits_{r=0}^{n} {}_nC_r = 16384.$

$$\sum_{r=0}^{n} {}_nC_r = 2^n$$

$$2^n = 16384$$

$$2^n = 2^{14}$$

$$n = 14.$$

15. $(25x^2 + 30yx + 9y^2)^3$

Using the AC method factor $25x^2 + 30y + 9y^2$

Sum = 30
Product = 225
factors (15, 15)

Substitute (15, 15) in the equation.

$25x^2 + 15yx + 15yx + 9y^2.$

factorize by grouping method

$(25x^2 + 15yx) + (15yx + 9y^2)$

$5x(5x + 3y) + 3y(5x + 3y)$

Collect the coefficients of each bracket.

$(5x+3y)(5x+3y) = (5x+3y)^2.$

Hence the initial equation becomes.

$((5x+3y)^2)^3 = (5x+3y)^6.$

Concider $(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6.$

$(a+b)^6 = (5x+3y)^6.$

Substitute $5x$ for $a$ and $3y$ for $b$.

$(5x+3y)^6 = (5x)^6 + 6(5x)^5(3y) + 15(5x)^4(3y)^2 + 20(5x)^3(3y)^3 + 15(5x)^2(3y)^4 + 6(5x)(3y)^5 + (3y)^6.$

$= 15625x^6 + 56250x^5y + 84375x^4y^2 + 67500x^3y^3 + 30375x^2y^4 + 7290xy^5 + 729y^6.$